

Radial basis function neural network control for parallel spatial robot

Nguyen Hong Quang¹, Nguyen Van Quyen², Nguyen Nhu Hien³

^{1,3}Thai Nguyen University of Technology, Vietnam

²Hanoi University of Science and Technology, Vietnam

Article Info

Article history:

Received Dec 9, 2019

Revised May 30, 2020

Accepted Jun 25, 2020

Keywords:

Inverse dynamics controller

Kronecker product

Numerical simulation

Parallel robot manipulator

RBF neural network control

ABSTRACT

The derivation of motion equations of constrained spatial multibody system is an important problem of dynamics and control of parallel robots. The paper firstly presents an overview of the calculating the torque of the driving stages of the parallel robots using Kronecker product. The main content of this paper is to derive the inverse dynamics controllers based on the radial basis function (RBF) neural network control law for parallel robot manipulators. Finally, numerical simulation of the inverse dynamics controller for a 3-RRR delta robot manipulator is presented as an illustrative example.

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Corresponding Author:

Nguyen Hong Quang,

Thai Nguyen University of Technology,

No. 666, 3/2 Street, Tich Luong Ward, Thai Nguyen City, Vietnam.

Email: quang.nguyenhong@tnut.edu.vn

1. INTRODUCTION

In the past three decades, the theory on dynamics of constrained multibody systems has been developed to a high degree of maturity [1-4]. The parallel robot manipulators are constrained multibody structures [5-7]. The equations of motion for a multibody system are obtained as the end result of a sequence of mathematical operators. In general, the known methods to derive the equations of motion of multibody systems are Lagrange's equations, Newton-Euler equations, Kane's equations. Among these methods, the approach using Lagrange's equations with multipliers has become an attractive method to derive the equations of motion of constrained multibody systems. This approach provides a well analytical and orderly structure that is very useful for control purposes.

The control of treelike multibody systems is of interest to a number of research communities in a very of applications areas. Many advanced methods for control of robot manipulators based on the Lagrange's equations have been developed [8-19]. The application of modern control methods such as sliding mode control method, the neural network control method for controller design of the treelike robot manipulators is presented in the works [20-30]. In contrast to the rapid progress in control theory of treelike robot manipulators, the development of the control theory for parallel robots is still limited. Modern control methods have also been used in the control problem of plane parallel manipulators [31-34]. One has used the control methods such as the proportional derivative (PD) control and proportional integral derivative (PID) control for designing some controllers of spatial parallel robot manipulators [35-38]. However, the application of modern control methods such as sliding mode control method, the radial basis function (RBF) neural network control method for controller design of the spatial parallel robot manipulators is a new problem that has not been investigated.

Recently, N. H. Quang [39-41] proposed a control method using model predictive approach. AL-Azzawi [42] address the control problem for a class of nonlinear dynamical systems based on linear feedback control strategies. S. Riache [43] proposed adaptive nonsingular terminal super-twisting controller consists of the hybridization of a nonsingular terminal sliding mode control and an adaptive super twisting. Simulations with nonsingular terminal super-twisting control to prove the superiority and the effectiveness of the proposed approach. In [44], a new compound hierarchical sliding mode control and fuzzy logic control scheme has been proposed for a class of underactuated systems with mismatched uncertainties.

In the present study, we present a control method using neural network for controller design of spatial parallel robot manipulators. In the section 2, the application of the new matrix form of Lagrangian equations with multipliers for constrained multibody systems to establish a new expression for calculation of the driving torques of parallel robots will be discussed. The inverse dynamics controller for the parallel robot manipulator is considered in the section 3. In the section 4, numerical simulation of the inverse dynamics controller for a 3-RRR delta parallel spatial robot manipulator is presented as an illustrative example.

2. INVERSE DYNAMICS OF CONSTRAINED MULTIBODY SYSTEMS

Let us consider a scleronomic multibody system of $f = n_a$ degree of freedom containing p rigid-bodies with r holonomic constraints. Let $s = [s_1, s_2, \dots, s_n]^T$ be the vector of generalized coordinates, the motion equations of constrained holonomic multibody systems can be written as:

$$M(s)\ddot{s} + C(s, \dot{s})\dot{s} + g(s) + \Phi_s^T(s)\lambda + d(s, \dot{s}) = \tau \quad (1)$$

$$f(s) = 0 \quad (2)$$

where $M(s)$ is the $n \times n$ mass matrix, $C(s, \dot{s})$ is the $n \times n$ coriolis/centripetal matrix, f is $r \times 1$ vector of constraint equations, $\Phi_s(s)$ is the $r \times n$ Jacobian matrix of the vector f , d is the $n \times 1$ vector of friction force and disturbance, τ is the $n \times 1$ vector of driving forces/torques, λ is the $r \times 1$ vector of Lagrangian multipliers. The Coriolis/Centripetal matrix $C(s, \dot{s})$ is determined from the mass matrix according the following formula [45, 46].

$$C(s, \dot{s}) = \frac{\partial M(s)}{\partial s} (E_n \otimes \dot{s}) - \frac{1}{2} \left[\frac{\partial M(s)}{\partial s} (\dot{s} \otimes E_n) \right]^T \quad (3)$$

The Jacobian matrix $\Phi_s(s)$ of the constrained equations is determined by the following formula;

$$\Phi_s = \frac{\partial f}{\partial s} = \begin{bmatrix} \frac{\partial f_1}{\partial s_1} & \dots & \frac{\partial f_1}{\partial s_n} \\ \dots & \dots & \dots \\ \frac{\partial f_r}{\partial s_1} & \dots & \frac{\partial f_r}{\partial s_n} \end{bmatrix} \quad (4)$$

Firstly, the generalized coordinates in vector s are divided into two subgroups: independent coordinates q_a , and redundant coordinates z . Then we have;

$$s = [q_a^T \quad z^T]^T, q_a = [q_1 \quad \dots \quad q_f]^T, z = [z_1 \quad \dots \quad z_r]^T, n = f + r \quad (5)$$

By differentiating in (2) with respect to vectors s, q_a, z , respectively, we obtain the following Jacobian matrices,

$$\Phi_s = \frac{\partial f}{\partial s} \in \mathbb{R}^{r \times n}, \Phi_z = \frac{\partial f}{\partial z} \in \mathbb{R}^{r \times r}, \Phi_a = \frac{\partial f}{\partial q_a} \in \mathbb{R}^{r \times f}, \Phi_s = [\Phi_a \quad \Phi_z] \quad (6)$$

By introducing the projection matrix [47]:

$$R(s) = \begin{bmatrix} E \\ -\Phi_z^{-1}\Phi_a \end{bmatrix} \in \mathbb{R}^{n \times f} \quad (7)$$

one has:

$$R^T(s)\Phi_s^T(s) = 0, \quad (8)$$

where E is the $f \times f$ identity matrix.

Left multiplication of the motion in (1) with the matrix R^T yields,

$$\begin{aligned} R^T [M(s)\ddot{s} + C(s, \dot{s})\dot{s} + g(s) + \Phi_s^T(s)\lambda + d(s, \dot{s})] &= R^T \begin{bmatrix} \tau_a \\ \tau_z \end{bmatrix} \\ &= [E \quad \Phi_z^{-1}\Phi_a(s)]^T \begin{bmatrix} \tau_a \\ \tau_z \end{bmatrix} \\ &= \tau_a - [\Phi_z^{-1}\Phi_a(s)]^T \tau_z \end{aligned} \quad (9)$$

where τ_a is the vector of the driving forces/torques in active joints and τ_z is the vector of the forces/torques in passive joints. Making use of in (8) and assuming that $\tau_z = 0$, the driving torques can be deduced from (9) as,

$$\tau_a = R^T [M(s)\ddot{s} + C(s, \dot{s})\dot{s} + g(s) + d(s, \dot{s})] \quad (10)$$

3. ADAPTIVE RBF NEURAL NETWORK CONTROL BASED ON INVERSE DYNAMICS FOR PARALLEL ROBOTS

3.1. Transformation of motion equations

To study the stability of the control algorithms, the motion equations of parallel robots are transformed into a suitable form. Let us consider a scleronomic constrained multibody system. From the constrained in (2) we get;

$$f(s) = f(q_a, z) = 0, \quad \dot{f} = \Phi_a \dot{q}_a + \Phi_z \dot{z} = 0 \quad (11)$$

Assuming that the Jacobian matrix Φ_z is nonsingular, $\det(\Phi_z) \neq 0$. From (11) one may obtain,

$$\dot{z} = -\Phi_z^{-1}\Phi_a \dot{q}_a \quad (12)$$

It is noted that,

$$q_a = E q_a \quad (13)$$

Combining (12) with (13) yields the following differential equation:

$$\dot{s} = R(s)\dot{q}_a \quad (14)$$

Differentiating in (14) with respect to time gives the acceleration relation as;

$$\ddot{s} = R(s)\ddot{q}_a + \dot{R}(s, \dot{s})\dot{q}_a = R(s)\ddot{q}_a + \frac{\partial R(s)}{\partial s}(E_p \otimes \dot{s})\dot{q}_a \quad (15)$$

Substituting in (14) and (15) into to (9) yields;

$$R^T(s) \left[M(s) \ddot{s} + \frac{\partial R(s)}{\partial s}(E_p \otimes \dot{s})\dot{q}_a + C(s, \dot{s})\dot{s} + g(s) + d(s, \dot{s}) \right] = \tau_a \quad (16)$$

To simplify the description, we define;

$$\begin{aligned} \bar{M}(s) &:= R^T(s)M(s)R(s)\bar{C}(s, \dot{s}) \\ &:= R^T(s) \left[M(s) \frac{\partial R(s)}{\partial s}(E_p \otimes \dot{s}) + C(s, \dot{s})R(s) \right] \bar{g}(s): \\ &= R^T(s)g(s)\bar{d}(s, \dot{s}):= R^T(s)d(s, \dot{s}) \end{aligned} \quad (17)$$

In (9) and (11) now can be rewritten as follows;

$$\bar{M}(s)\ddot{q}_a + \bar{C}(s, \dot{s})\dot{q}_a + \bar{g}(s) + \bar{d}(s, \dot{s}) = \tau_a \quad (18)$$

$$f(s) = 0 \quad (19)$$

The motion equations of parallel robots (18) and (19) are called the motion equations in mixture form. Where s is the vector of redundant generalized coordinates and q_a is the vector of independent coordinates.

We will use this equation as the basis for designing the controller for parallel robots. For this purpose, we prove the following properties [33]:

- \bar{M} is a symmetric positive definite matrix: $\bar{M}^T = \bar{M}$,
- $\dot{\bar{M}} - 2\bar{C}$ is a skew-symmetric matrix: $(\dot{\bar{M}} - 2\bar{C})^T = -(\dot{\bar{M}} - 2\bar{C})$.

Due to the symmetry of the matrix M is symmetric, one has;

$$\bar{M}^T(s) = [R^T(s)M(s)R(s)]^T = R^T(s)M(s)R(s) = \bar{M}(s)$$

Since $M(s)$ is positive definite, $\bar{M}(s)$ is also a positive definite matrix. Using the (17), one obtains;

$$\begin{aligned} \dot{\bar{M}}(s) - 2\bar{C}(s, \dot{s}) &= \dot{R}^T M R + R^T \dot{M} R + R^T M \dot{R} - 2R^T (M \dot{R} + C R) \\ &= \dot{R}^T M R + R^T \dot{M} R - R^T M \dot{R} - 2R^T C R \\ &= R^T (\dot{M} - 2C) R - R^T M \dot{R} + \dot{R}^T M R \end{aligned} \quad (20)$$

Since $\dot{M} - 2C$ is skew-symmetric [8], from in (20) one has;

$$\begin{aligned} [\dot{\bar{M}}(s) - 2\bar{C}(s, \dot{s})]^T &= [R^T (\dot{M} - 2C) R]^T - (R^T M \dot{R})^T + (\dot{R}^T M R)^T \\ &= -R^T (\dot{M} - 2C) R - \dot{R}^T M R + R^T M \dot{R} = -[\dot{\bar{M}}(s) - 2\bar{C}(s, \dot{s})] \end{aligned} \quad (21)$$

Thus, $\dot{\bar{M}}(s) - 2\bar{C}(s, \dot{s})$ is a skew symmetric matrix.

3.2. RBF neural network control law and stability analysis

In practice, the perfect robot model could be difficult to obtain, and external disturbances are always present in practice. The uncertain motion equations of parallel robots with $n_a = f$ active joints (18) can be described in the following form;

$$\bar{M}(s)\ddot{q}_a + \hat{C}(s, \dot{s})\dot{q}_a + \hat{g}(s) + \hat{d}(s, \dot{s}) = \tau_a \quad (22)$$

where $\bar{M}(s)$ is an $f \times f$ inertia matrix, $\hat{C}(s, \dot{s})$ is an $f \times f$ matrix containing the centrifugal and Coriolis terms, $\hat{g}(s)$ is an $f \times 1$ vector containing gravitational forces and torques, s is the vector of generalized coordinates, q_a is active joint coordinates, and \hat{d} denotes disturbances. It is supposed that,

$$\begin{aligned} \bar{M}(s) &= \bar{M}(s) + \Delta\bar{M}(s) \\ \hat{C}(s, \dot{s}) &= \bar{C}(s, \dot{s}) + \Delta\bar{C}(s, \dot{s}) \\ \hat{g}(s) &= \bar{g}(s) + \Delta\bar{g}(s) \\ \hat{d}(s, \dot{s}) &= \bar{d}(s, \dot{s}) + \Delta\bar{d}(s, \dot{s}) \end{aligned} \quad (23)$$

where \bar{M} , \bar{C} , \bar{g} , \bar{d} are the prior-known components and $\Delta\bar{M}$, $\Delta\bar{C}$, $\Delta\bar{g}$, $\Delta\bar{d}$ are modeling errors of \bar{M} , \bar{C} , \bar{g} and \bar{d} respectively. Assume that the modeling errors are bounded by some finite constants as;

$$\|\Delta\bar{M}\| \leq m_0, \quad \|\Delta\bar{C}\| \leq c_0, \quad \|\Delta\bar{g}\| \leq g_0, \quad \|\Delta\bar{d}\| \leq d_0 \quad (24)$$

where m_0 , c_0 , g_0 , d_0 are known constants. Substituting in (23) into to (22) yields.

$$(\bar{M} + \Delta\bar{M})\ddot{q}_a + (\bar{C} + \Delta\bar{C})\dot{q}_a + \bar{g} + \Delta\bar{g} + \bar{d} + \Delta\bar{d} = \tau_a \quad (25)$$

From (25) one has;

$$\bar{M}(s)\ddot{q}_a + \bar{C}(s, \dot{s})\dot{q}_a + \bar{g}(s) + \bar{d}(s, \dot{s}) + \bar{h}(s, \dot{s}) = \tau_a \quad (26)$$

where $\bar{h}(s, \dot{s})$ is the sum of unknown terms of the dynamic system.

$$\bar{h}(s, \dot{s}) = \Delta \bar{M} \ddot{q}_a + \Delta \bar{C} \dot{q}_a + \Delta \bar{g} + \Delta \bar{d} \quad (27)$$

Assume that $\|\bar{h}(s, \dot{s})\| \leq h_0$. The sliding mode function is selected as;

$$n(t) = \dot{e}_a(t) + \Lambda e_a(t) \quad (28)$$

where Λ is the positive diagonal matrix.

$$\Lambda = \text{diag } \lambda_1, \lambda_2, \dots, \lambda_{n_a} \quad , \quad \lambda_i > 0 \quad ; \quad i = 1, 2, \dots, n_a \quad (29)$$

In (28) we define,

$$e_a(t) = q_a(t) - q_a^d(t) \quad (30)$$

where $q_a^d(t)$ is the vector of desired trajectory and $q_a(t)$ is the vector of real trajectory. The function $\bar{h}(s, \dot{s})$ can be rewritten as:

$$\bar{h}(n) = \bar{h}(s, \dot{s}) \quad (31)$$

The function $\bar{h}(n)$ is the main reason for the degradation of the control quality. If this effect is compensated, the control accuracy can then be improved. According to Stone-Weierstrass theorem [23-24] one can choose an appropriate artificial neural network (ANN) with a limited number of neurons that can approximate an unknown nonlinear function with a given accuracy. For approximating function $\bar{h}(n)$ we choose the following simple structure ANN:

$$\bar{h}(n) = Ws + e = \hat{h}(n) + e \quad (32)$$

where W is the $n_a \times n_a$ matrix, $\hat{h}(n) = [\hat{h}_1, \hat{h}_2, \dots, \hat{h}_{n_a}]^T = Ws$ is the approximation of $\bar{h}(n)$, e is the approximation error. If $\|\bar{h}(n)\| \leq h_0$, we have $\|e\| \leq \varepsilon_0$. Assuming that the matrix W has n_a column vectors w_i , we have;

$$\hat{h} = [\hat{h}_1, \hat{h}_2, \dots, \hat{h}_{n_a}]^T = W\sigma = \sum_{i=1}^{n_a} \sigma_i w_i \quad (33)$$

In this paper, the radial basis function (RBF) neural network was used as shown in Figure 1. This structure has been proved to satisfy the Stone-Weierstrass theorem [23]. If we choose the Gaussian activation function σ_i according to the formula,

$$\sigma_i = \exp \left[-\frac{\|v - c_i\|^2}{\chi_i^2} \right] \quad (34)$$

where the vector c_i represents the coordinate value of the center point of the Gaussian function of neural net i , and χ_i is derivation parameter which is freely chosen, the function approximation \hat{h} has the following form;

$$\hat{h}_i = \sum_{j=1}^{n_a} \sigma_j w_{ji} \quad , \quad i = 1, \dots, n_a \quad (35)$$

where w_{ji} are the weights to be updated of the approximating neural network.

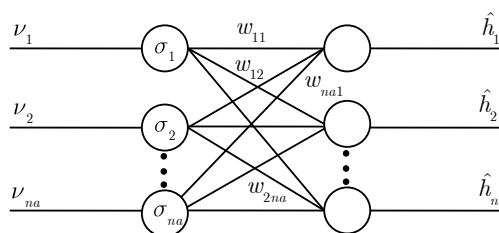


Figure 1. RBF neural network structure

The control problem is now to find the control torque u and learning algorithm of w_{ji} of the neural network so that $n \rightarrow 0$ and position errors $e \rightarrow 0$, granting $q_a(t) \rightarrow q_a^d(t)$.

Theorem: The trajectory $q_a(t)$ of dynamic system defined by (3.34) with RFB neural network according to (3.60), (3.62), and the sliding surface (3.40) will track the desired trajectory $q_a^d(t)$ with error $e(t) = q_a(t) - q_a^d(t) \rightarrow 0$ if the control law u the learning algorithm w_i are chosen as follows;

$$u = \bar{M}(s)\ddot{q}_a^d + \bar{C}(s, \dot{s})\dot{q}_a^d + \bar{g} + \bar{d} - \bar{M}(s)\Lambda\dot{e}_a - \bar{C}(s, \dot{s})\Lambda e_a - Kn - \gamma \frac{n}{\|n\|} + (1 + \eta)Ws \quad (36)$$

$$\dot{w}_i = -\eta\sigma_i n \quad (37)$$

where K is a $n_a \times n_a$ symmetric positive matrix, and $\eta > 0, \gamma > 0$. Noting that the states q_a, \dot{q}_a in the control law (36) are measured.

Proof: This theorem can be proved using the Lyapunov direct method. We choose the Lyapunov function as;

$$V(t) = \frac{1}{2} [n^T \bar{M} n + \sum_{i=1}^{n_a} w_i^T w_i] \quad (38)$$

Since $\bar{M}(s)$ is symmetric and positive definite $V(t) > 0$ for $n \neq 0, w_i \neq 0$ and $V(t) = 0$ if and only if $n = 0, w_i = 0$.

The derivative of the function $V(t)$ is,

$$\dot{V}(t) = n^T \bar{M} \dot{n} + \frac{1}{2} n^T \dot{\bar{M}} n + \sum_{i=1}^{n_a} w_i^T \dot{w}_i \quad (39)$$

Using the skew-symmetry of the matrix $\bar{M}(s) - 2\bar{C}(s, \dot{s})$, we have

$$n^T (\dot{\bar{M}} - 2\bar{C}) n = 0 \rightarrow n^T \dot{\bar{M}} n = 2n^T \bar{C} n \quad (40)$$

Substituting (40) into (39) gives:

$$\dot{V}(t) = n^T (\bar{M} \dot{n} + \bar{C} n) + \sum_{i=1}^{n_a} w_i^T \dot{w}_i \quad (41)$$

If we choose $u = \tau_a$, from (38) and (26) one has,

$$\bar{M} \dot{n} + \bar{C} n = - \left[Kn + \gamma \frac{n}{\|n\|} - (1 + \eta)Ws + \bar{h}(n) \right] \quad (42)$$

Substituting in (42) into to (41) yields,

$$\dot{V}(t) = n^T \left[-Kn - \gamma \frac{n}{\|n\|} + \eta W \sigma - e \right] + \sum_{i=1}^{n_a} w_i^T \dot{w}_i \quad (43)$$

Using the learning algorithm (37), the last term in (43) has the following form,

$$\sum_{i=1}^{n_a} w_i^T \dot{w}_i = -\eta \sum_{i=1}^{n_a} w_i^T n \sigma_i = -\eta n^T w s \quad (44)$$

Substituting (44) into to (43) yields,

$$\dot{V}(t) = -n^T Kn - \gamma \frac{n^T n}{\|n\|} - n^T e \quad (45)$$

If we select $\gamma = \delta + \varepsilon_0$, and $\delta > 0$, one obtains: $\dot{V}(t) = -n^T Kn - \delta \|n\| - (\varepsilon_0 \|n\| + n^T e)$ (46)

Since $\|e\| < \varepsilon_0$, $\dot{V}(t) < 0$ for all $n \neq 0$, and $\dot{V}(t) = 0$ if and only if $n = 0$. It follows from Lyapunov's theory that the system is asymptotically stable, or $n \rightarrow 0$ as $t \rightarrow \infty$, therefore,

$$e_a(t) = q_a(t) - q_a^d(t) \rightarrow 0 \quad (47)$$

4. SIMULATION EXAMPLE

From the RBF neural network control law presented in the above section, the resulting block scheme is illustrated in the Figure 2. For the use of the control law u based on (36) and (37), actual signals $q_a, \dot{q}_a, \ddot{q}_a$ are assumed to be known. Therefore, one can obtain actual values of generalized coordinates, velocities and accelerations s, \dot{s}, \ddot{s} . The numerical simulation may be a possible way to suggest an alternative choice for actual values of generalized coordinates, velocities and accelerations. Considering the dynamic equations of parallel robot manipulator as;

$$M(s)\ddot{s} + C(s, \dot{s})\dot{s} + g(s) + \Phi_s^T(s)\lambda + d(s, \dot{s}) = \tau \quad (48)$$

$$f(s) = 0 \quad (49)$$

where,

$$\tau = \tau(t, s, \dot{s}, s^d, \dot{s}^d) = [u^T, 0^T]^T \quad (50)$$

Differentiating in (49) with respect to time gives,

$$\dot{f}(s) = \frac{\partial f}{\partial s} \dot{s} = \Phi_s \dot{s} = 0 \quad (51)$$

$$\ddot{f}(s, \dot{s}) = \Phi_s \ddot{s} + \dot{\Phi}_s \dot{s} = 0 \quad (52)$$

where [45, 46]:

$$\dot{\Phi}_s(s) = \frac{\partial \Phi}{\partial s} (E_n \otimes \dot{s}) \quad (53)$$

Define,

$$p_1(s, \dot{s}, s^d, \dot{s}^d, t) = \tau - C(s, \dot{s})\dot{s} - g(s) - d(s, \dot{s}) \quad (54)$$

$$p_2(s, \dot{s}, t) = -\dot{\Phi}_s(s)\dot{s} = -\left[\frac{\partial \Phi_s}{\partial s} (E_n \otimes \dot{s})\right] \dot{s} \quad (55)$$

In (48) and (52) now can be written in the following form,

$$M(s)\ddot{s} + \Phi_s^T(s)\lambda = p_1(s, \dot{s}, s^d, \dot{s}^d, t) \quad (56)$$

$$\Phi_s(s)\ddot{s} = p_2(s, \dot{s}, t) \quad (57)$$

Left multiplication of (56) with the matrix R^T yields,

$$R^T(s)M(s)\ddot{s} + R^T(s)\Phi_s^T(s)\lambda = R^T(s)p_1(s, \dot{s}, t) \quad (58)$$

According to (8), (58) becomes:

$$R^T(s)M(s)\ddot{s} = R^T(s)p_1(s, \dot{s}, s^d, \dot{s}^d, t) \quad (59)$$

In (59) is a system of f second-order differential equations. Combining in (59) with (57) yields,

$$\begin{bmatrix} R^T(s)M(s) \\ \Phi_s(s) \end{bmatrix} \ddot{s} = \begin{bmatrix} R^T(s)p_1 \\ p_2 \end{bmatrix} \quad (60)$$

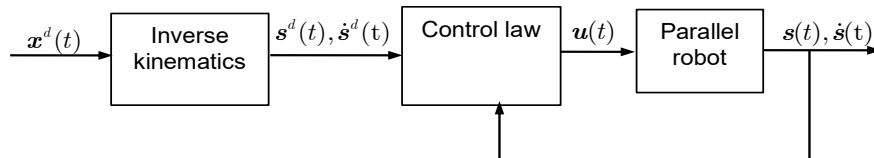
If the matrix,

$$A(s) = \begin{bmatrix} R^T(s)M(s) \\ \Phi_s(s) \end{bmatrix} \quad (61)$$

is nonsingular, from (60) one obtains the following differential equation system,

$$\ddot{s} = \ddot{s}(s, \dot{s}, s^d, \dot{s}^d, t) \quad (62)$$

Then, solving the (62) we find s, \dot{s} [48]. Therefore we can calculate control law according to (36).



A 3-RRR spatial parallel robot manipulator shown in Figure 3 is utilized in this study to verify the effectiveness of the proposed control scheme. The mechanical model for the 3-RRR delta robot manipulator is a system of rigid bodies connected by joints as Figure 4. The parallelogram mechanisms that connect the driving links to the mobile platform are modeled as homogeneous rods with universal and spherical joints at two ends. From Figures 4 and 5 it is followed that the configuration of the 3-RRR delta spatial parallel robot manipulator is represented by a vector of generalized coordinates as:

$$\mathbf{s} = [\psi_1 \ \psi_2 \ \psi_3 \ \gamma_1 \ \gamma_2 \ \gamma_3, x_P \ y_P \ z_P]^T$$

The differential-algebraic equations of the system are given in the Appendix. The kinematic and dynamic parameters of the robot manipulator are given in the Table 1. In the simulation, the center of the moving platform will be controlled to track the given trajectory defined by,

$$x_p = 0.3 \cos 2\pi t ; y_p = 0.3 \sin 2\pi t ; z_p = -0.7 \text{ (m)}$$

The parameters of the neural network control law are chosen as follows,

$$\mathbf{K} = \text{diag } 80, 80, 80; \mathbf{L} = \text{diag}(80, 80, 80); \eta = 1.1; \gamma = 200;$$

$$\chi_1 = 1; \chi_2 = 2; \chi_3 = 3; c_1 = 0.01; c_2 = 0.02; c_3 = 0.03;$$

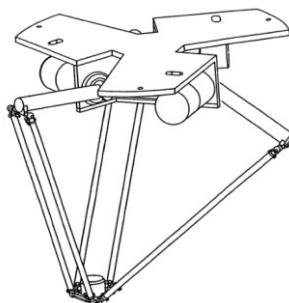


Figure 3. Delta robot with three parallelogram mechanisms

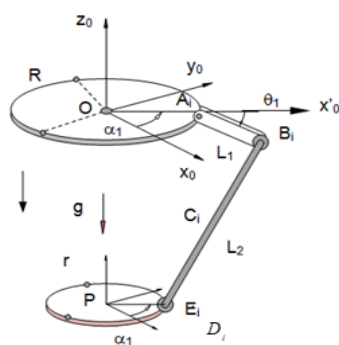


Figure 4. Model of 3-RRR Delta robot

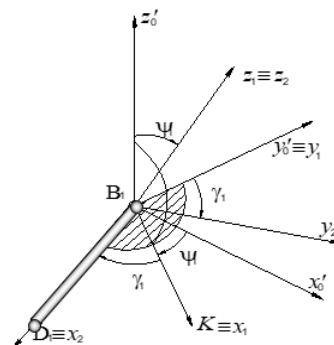


Figure 5. Position of the $B_i D_i$ rod in the space

Table 1. The parameters of the Delta robot in Figure 3

L_1	L_2	R	r	r	α_1	α_1	α_2	α_3	m_1	m_1	m_2	m_2	m_p
0.3 (m)	0.8 (m)	0.266 (m)	0.04 (m)	0 (rad)	$\frac{2\pi}{3}$ (rad)	$\frac{4\pi}{3}$ (rad)	$\frac{4\pi}{3}$ (rad)	0.42 (kg)	2 · 0.2 (kg)	0.75 (kg)			

In this paper the modeling errors are chosen to be 20% of the prior-known values of the nominal model as,

$$\Delta M(s) = 20\%M(s); \Delta C(s, \dot{s}) = 20\%C(s, \dot{s}); \Delta g(s) = 20\%g(s)$$

The disturbance vector is chosen as $\mathbf{d} = [\kappa_1 \sin 20t \ \kappa_1 \cos 20t \ \dots \ \kappa_6 \sin 20t \ \kappa_6 \cos 20t]^T$. Some simulation results are given in the Figures from 6 to 9. The position errors of the moving platform are shown in Figures 6 and 7. The control torques are shown in Figures 8 and 9. The stationary errors in position of the platform are kept about 10^{-4} mm.

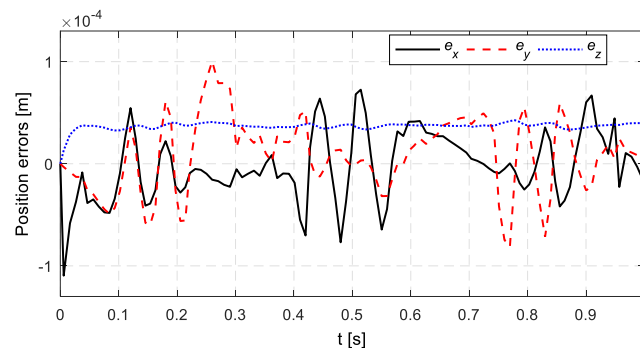


Figure 6. Position errors of the moving platform without the modeling errors and disturbance

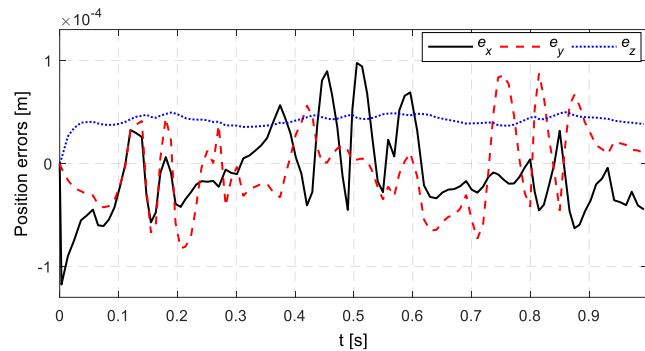


Figure 7. Position errors of the moving platform with the modeling errors and disturbance

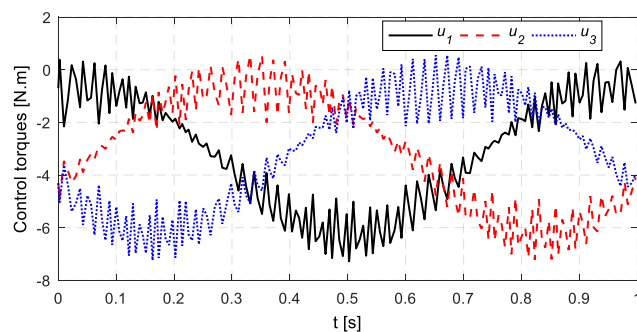


Figure 8. Control torques without the modeling errors and disturbance

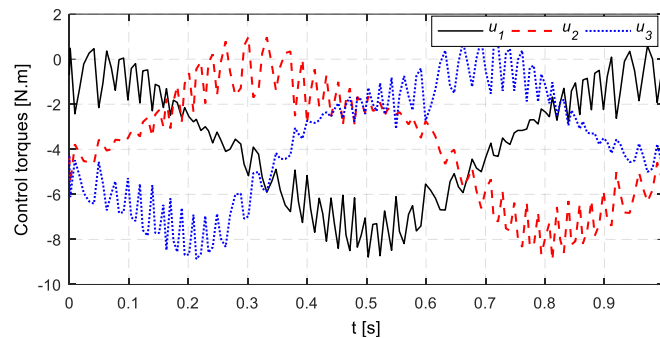


Figure 9. Control torques with the modeling errors and disturbance

5. CONCLUSIONS

Many modern methods for control of robot manipulators based on the Lagrangian multipliers have been developed. In contrast to the rapid progress in control theory of treelike robot manipulators, the development of the modern control theory for parallel robot manipulators is still limited. This paper presented the application of the RBF neural network control law to compensate uncertainties in the parallel robot manipulators. The new matrix form of Lagrangian equations with multipliers for constrained multibody systems was used to derive dynamic equations of spatial parallel robot manipulators based on computer software packages. Using adaptive RBF neural network control method, the controller for spatial robot manipulators based on inverse dynamics was developed. The stability of the control law using adaptive RBF neural network method for the control problem of spatial robot manipulators based on inverse dynamics was proven. Using Simulink program, the numerical simulation of the adaptive RBF neural network controller for a 3-PRR spatial parallel robot manipulator is studied. The application of modern methods for motion control of the constrained spatial multibody systems and spatial parallel robot manipulators will be presented in other works.

ACKNOWLEDGEMENTS

This research was supported by Research Foundation funded by Thai Nguyen University of Technology.

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